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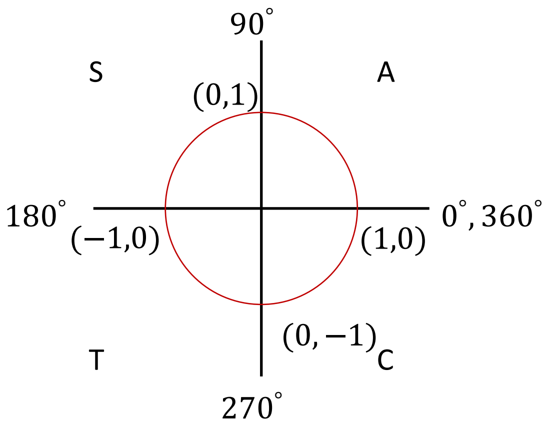
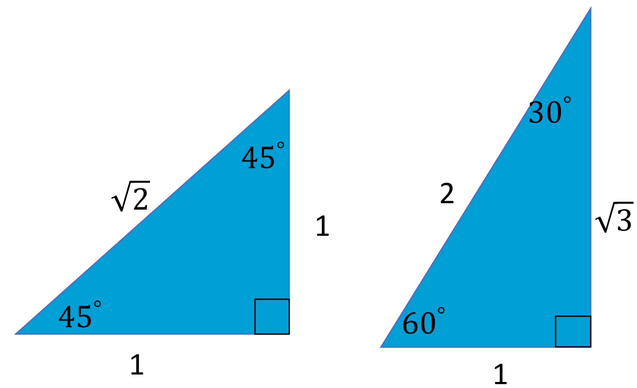
Algebra 2/Trig. Cheat Sheet

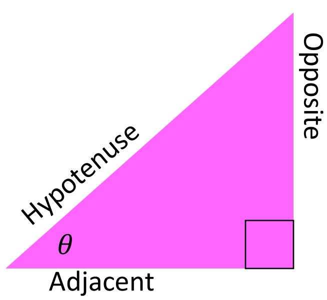
Unit Circle:



Special Triangles:



Trigonometric Functions: (SOH CAH TOA)
(Only work on right triangles.)



$$sin(\theta) = \frac{opposite}{hypotenuse}$$

$$cos(\theta) = \frac{adjacent}{hypotenuse}$$

$$tan(\theta) = \frac{opposite}{adjacent}$$

Trigonometric Inverses/Identities:

$$csc(\theta) = \frac{1}{sin(\theta)} \qquad sec(\theta) = \frac{1}{cos(\theta)}$$

$$cot(\theta) = \frac{cos(\theta)}{sin(\theta)} \qquad cot(\theta) = \frac{1}{tan(\theta)}$$

$$tan(\theta) = \frac{sin(\theta)}{cos(\theta)}$$

Co-function Identities:

$$sin\left(\frac{\pi}{2} - \theta\right) = cos(\theta) \qquad csc\left(\frac{\pi}{2} - \theta\right) = sec(\theta) \qquad tan\left(\frac{\pi}{2} - \theta\right) = cot(\theta)$$

$$cos\left(\frac{\pi}{2} - \theta\right) = sin\theta \qquad sec\left(\frac{\pi}{2} - \theta\right) = csc(\theta) \qquad cot\left(\frac{\pi}{2} - \theta\right) = tan(\theta)$$

Pythagorean Identities:

$$sin^2\theta + cos^2\theta = 1$$

$$1 + tan^2\theta = sec^2\theta$$

$$1 + cot^2\theta = csc^2\theta$$

Trig. Proofs:

$$tan\theta = sin\theta sec\theta$$

$$tan\theta = sin\theta \times \frac{1}{cos\theta}$$

$$tan\theta = \frac{sin\theta}{cos\theta}$$

$$tan\theta = tan\theta$$

Converting Degrees to Radians:

$$60^\circ \times \frac{\pi}{180} = \frac{\pi}{3}$$

Converting Radians to Degrees:

$$\frac{\pi}{3} \times \frac{180}{\pi} = 60^\circ$$

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Distance Formula:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Law of Cosines: $c^2 = a^2 + b^2 - 2abcos(c)$

Product of Roots: $\frac{c}{a}$

Sum of Roots: $\frac{-b}{a}$

Law of Sines:
(This will be included on reference table)

$$\frac{sin(A)}{a} = \frac{sin\ (B)}{b} = \frac{sin(C)}{c}$$

OR

$$\frac{a}{sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Solving Trig. Functions Algebraically:
Solve for θ, in the interval $-2\pi < \theta < 2\pi$

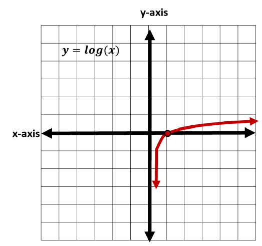
$$sin^2\theta - 1 = 0$$
$$(sin\theta + 1)(sin\theta - 1) = 0$$

$sin\theta + 1 = 0$	$sin\theta - 1 = 0$
$sin\theta = -1$	$sin\theta = 1$
$sin^{-1}(-1) =$	$sin^{-1}(1) =$
$\theta = -90^\circ, 270^\circ$	$\theta = -270^\circ, 90^\circ$

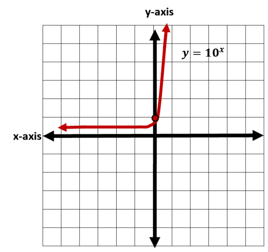
$$\theta = -270^\circ, -90^\circ, 90^\circ, 270^\circ$$

Logarithms are Inverses of Exponential Equations:

$y = \log_{10}(x)$



$y = 10^x$



Logarithm Rules:
$log_{10}(100) = 2 \longrightarrow 10^2 = 100$
$\log(ab) = \log(a) + \log\ (b)$
$\log\left(\frac{a}{b}\right) = \log(a) - \log\ (b)$
$\log a^b = b \cdot log(a)$
$ln_e(x) = e^x$

To Change Log Base:
$log_a(x) = \frac{lob_b(x)}{log_b(a)}$

Logarithmic Equations:

$$log_4(x) + log_4(x+6) = 2$$
$$log_4(x(x+6)) = 2$$
$$log_4(x^2 + 6x) = 2$$
$$4^2 = x^2 + 6x$$
$$x^2 + 6x - 16 = 0$$
$$(x+8)(x-2) = 0$$

$x + 8 = 0$	$x - 2 = 0$
$x = -8$	$x = 2$

$$x = -8{,}2$$

Radical Equations:
$$\frac{10\sqrt{x+4}}{10} = \frac{100}{10}$$
$$\sqrt{x+4} = 10$$
$$(\sqrt{x+4})^2 = 10^2$$
$$x + 4 = 100$$
$$x = 96$$

Simplifying Radical Expressions:
$\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$

$\sqrt[4]{256m^4n^8} = 4mn^2$

$25^{-\frac{1}{2}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$

Rationalizing Radical in denominator:
$$\frac{1}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{x}$$

Radical Rules:
$\sqrt[n]{x} = x^{\frac{1}{n}}$

$x^{\frac{m}{n}} = (\sqrt[n]{x})^m$

Imaginary Numbers:

$i^0 = 1$
$i^1 = i$
$i^2 = -1$
$i^3 = -i$
$a + bi$

Given i^{64} divide by 4 using long division, get remainder 0, then use as exponent.
$i^{64} \rightarrow i^0 = 1$

Sequences:

Arithmetic Sequence: (add or subtract)

$$a_n = a_1 + (n-1)d$$

Geometric Sequence: (multiply or divide)

$$a_n = a_1 \cdot r^{n-1}$$

Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Equation of a Circle:

$$(x-4)^2 + (y+2)^2 = 16$$

Center= (4, -2)

Radius= $\sqrt{16} = 4$

Probability:

Binomial Probability= ${}_nC_r \cdot p^r \cdot q^{n-r}$

Permutation= ${}_nP_r = \dfrac{n!}{(n-r)!}$
(Order matters)

Combinations= ${}_nC_r = \dfrac{n!}{r!(n-r)!}$
(Order doesn't matter)

Conditional= $\dfrac{P(A \cap B)}{P(A)}$=P$(B|A)$

Dependent Events= $P(A) \cdot P(\frac{B}{A})$
P (A and B)

Independent Events=$P(A) \cdot P(B)$
P (A and B)

Not mutually Exclusive= $P(A) + P(B) - P\ (A)$
P (A or B)

Mutually Exclusive=$P(A) + P\ (B)$
P (A or B)

Population Mean (μ):

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Population Standard Deviation (σ):

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x-\mu)^2}{N}}$$

Z-Score:

$$z = \frac{x-\mu}{\sigma}$$

Sample Mean ($\bar{x}$):

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

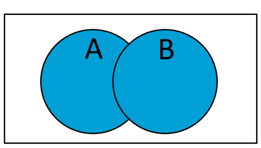
Sample Standard Deviation (S):

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x-\bar{x})^2}{n-1}}$$

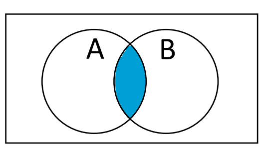
Margin of Error:

$$MOE = z \times \frac{s}{\sqrt{n}}$$

Venn Diagram:



$A \cup B$



$A \cap B$

Absolute Value: When solving for *x* create two equations. One with a positive answer, one with a negative. Then solve as normal. Ex:

$$|-125 + 2x| = 45$$

$-125 + 2x = \mathbf{45}$
$x = 85$

$-125 + 2x = \mathbf{-45}$
$x = 40$

Factoring Methods to Know:

1) Greatest Common Factor (GCF)
2) Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
2) Completing the Square
3) Difference of Two Squares (DOTS)
4) Factor by Grouping

Binomial Expansion:

$a^2 - b^2 = (a-b)(a+b)$
$a^3 + b^3 = (a+b)(a^2 + 2ab + b^2)$
$a^3 - b^3 = (a-b)(a^2 - 2ab + b^2)$

Discriminant Rules:

$b^2 - 4ac$

$b^2 - 4ac > 0$	2 real roots
$b^2 - 4ac = 0$	1 real root
$b^2 - 4ac < 0$	2 imaginary roots

Dividing Polynomials:

$$
\begin{array}{r l}
 & \quad 3x^2 + x + 1 \\
2x+1 & \overline{)\, 6x^3 + 5x^2 + 3x + 4} \\
- & \underline{\;\; 6x^3 + 3x^2} \\
 & \qquad\quad 2x^2 + 3x + 4 \\
 & - \quad \underline{2x^2 + x} \\
 & \qquad\qquad\quad 2x + 4 \\
 & \qquad - \quad \underline{2x + 1} \\
 & \qquad\qquad\quad R: 3
\end{array}
$$

$$Answer: 3x^2 + x + 1 + \frac{3}{2x+1}$$

Standard Form of a Parabola:

$y = ax^2 + bx + c$

Axis of Symmetry: $-\frac{b}{2a}$

Vertex Form of a Parabola:

$y = a(x-h)^2 + k$

Vertex: (h, k)

$0 < a < 1$, then parabola gets bigger

$a > 0$, then parabola gets narrow

Remainder Theorem: If polynomial $f(x)$ is divided by $(x - a)$, then remainder is equal to $f(a)$. Ex:

If $f(x) = 4x^2 + 6x + 15$ is divided by $(x - 1)$

The remainder will be…
$f(1) = 4(1)^2 + 6(1) + 15 = 25$

Simplifying Rational Expressions:

$$\frac{9y^3 - y}{3y - 1} = \frac{y(9y^2 - 1)}{3y - 1} = \frac{y\cancel{(3y-1)}(3y+1)}{\cancel{3y-1}} = y(3y+1)$$

Compound Interest Formula:

$$A = P(1 + \frac{r}{n})^{nt}$$

$P = Principle$
$r = Interest\ rate$
$n = number\ of\ compoundings\ per\ year$
$t = Total\ number\ of\ years$

Exponential Equation:

$y = ab^x$

Exponential Growth:

$b > 1$

Exponential Decay:

$0 < b < 1$

Power Functions:

$$y = kx^n$$

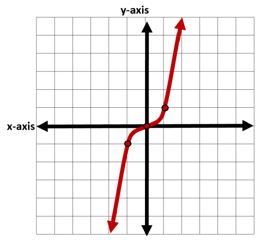
where, k and n are known number constants.

Odd Power Functions: $y = kx^n$

where n is equal to any odd number $\{1,3,5,7, \ldots \ldots \ldots\}$

Ex:

$y = x^3$

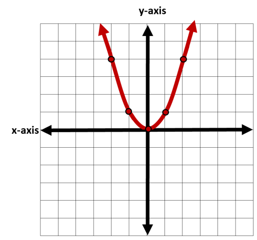


Even Power Functions: $y = kx^n$

where n is equal to any even number $\{2, 4, 6, 8, \ldots \ldots \ldots\}$

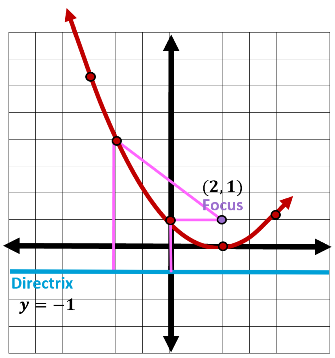
Ex:

$y = x^2$



Focus and Directrix:

A parabola is equi-distant between both focus and directrix.



To find an equation of parabola, given focus $(2,1)$ and directrix $y = -1$:

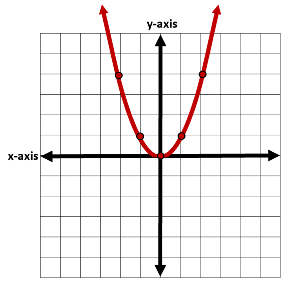
$$\sqrt{(y - directrix)^2} = \sqrt{(x-a)^2 + (y-b)^2}$$

(where a and b are from focus)

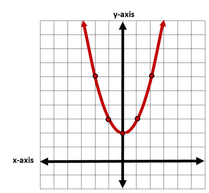
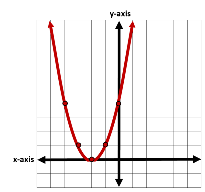
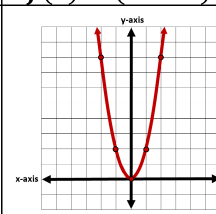
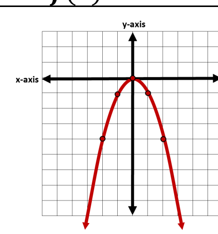
$$\sqrt{(y+1)^2} = \sqrt{(x-2)^2 + (y-1)^2}$$

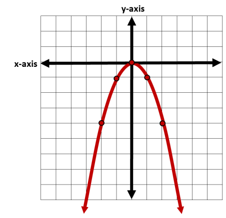
$$y = \frac{(x-2)^2}{4}$$

Transformations of Function $f(x) = x^2$, where $C = 2$:



$f(x) = x^2$

Function Transformation	What does it do to the graph?	Graph
$y = f(x) + C$	$C > 0$ *moves up* $C < 0$ *moves down*	 $f(x) = x^2 + 2$
$y = f(x + C)$	$C > 0$ *moves left* $C < 0$ *moves right*	 $f(x) = (x^2 + 2)$
$y = Cf(x)$	$C > 1$ *moves closer to* $y - axis$ $0 < C < 1$ *moves further from* $y - axis$	 $f(x) = 2x^2$
$y = -f(x)$	*Reflection in the* $x - axis$	 $f(x) = -x^2$

$\boldsymbol{y = f(-x)}$	$Reflection\ in\ the\ y - axis$	 $\boldsymbol{f(x) = -x^2}$

Pre-Calculus Reference Sheet

Factoring

- $a^2 - b^2 = (a-b)(a+b)$
- $a^2 + b^2$ is prime
- $a^2 + 2ab + b^2 = (a+b)^2$
- $a^2 - 2ab + b^2 = (a-b)^2$
- $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$
- $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$

Analytic Geometry

- slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- equation of a line: $y - y_1 = m(x - x_1)$
- distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Exponent Rules

- $a^{x+y} = a^x a^y$
- $(ab)^x = a^x b^x$
- $(a^x)^y = a^{xy}$
- $a^0 = 1$ if $a \neq 0$
- $a^{-x} = \dfrac{1}{a^x}$ if $a \neq 0$
- $a^{x-y} = \dfrac{a^x}{a^y}$ if $a \neq 0$

Logarithm Rules

- $\log_b x = y \iff x = b^y$
- $b^{\log_b x} = x$
- $\log_b b^x = x$
- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b xy = \log_b x + \log_b y$
- $\log_b \dfrac{x}{y} = \log_b x - \log_b y$
- $\log_b x^y = y \log_b x$
- $\log_b x = \dfrac{\log_a x}{\log_a b}$

Arithmetic Series

- $a_k = a + (k-1)d$
- $S_n = \displaystyle\sum_{k=1}^{n} [a + (k-1)d] = \frac{n}{2}[2a + (n-1)d]$
- $S_n = \displaystyle\sum_{k=1}^{n} [a + (k-1)d] = n\left(\frac{a + a_n}{2}\right)$

Geometric Series

- $a_n = ar^{n-1}$
- $S_n = \displaystyle\sum_{k=0}^{n-1} ar^k = a\left[\frac{1-r^n}{1-r}\right]$ if $r \neq 1$
- $S = \displaystyle\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$ if $|r| < 1$

Trigonometry

- $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ • $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$
- $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$

	$0°$	$30°$	$45°$	$60°$	$90°$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Pythagorean Identities

- $\cos^2 A + \sin^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $1 + \cot^2 A = \csc^2 A$

Ratio Identities

- $\tan A = \dfrac{\sin A}{\cos A}$ • $\cot A = \dfrac{\cos A}{\sin A}$

Reciprocal Identities

- $\sec A = \dfrac{1}{\cos A}$ • $\csc A = \dfrac{1}{\sin A}$ • $\cot A = \dfrac{1}{\tan A}$

Sum and Difference Identities

- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Double Angle Identities

- $\cos 2A = \cos^2 A - \sin^2 A$
- $\cos 2A = 2\cos^2 A - 1$
- $\cos 2A = 1 - 2\sin^2 A$
- $\sin 2A = 2\cos A \sin A$
- $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

Half Angle Identities

- $\cos \dfrac{A}{2} = \pm\sqrt{\dfrac{1 + \cos A}{2}}$
- $\sin \dfrac{A}{2} = \pm\sqrt{\dfrac{1 - \cos A}{2}}$
- $\tan \dfrac{A}{2} = \dfrac{1 - \cos A}{\sin A} = \dfrac{\sin A}{1 + \cos A}$

Triple Angle Identities

- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\sin 3A = 3\sin A - 4\sin^3 A$

Power Reduction Identities

- $\cos^2 A = \dfrac{1 + \cos 2A}{2}$
- $\sin^2 A = \dfrac{1 - \cos 2A}{2}$
- $\tan^2 A = \dfrac{1 - \cos 2A}{1 + \cos 2A}$
- $\cos^3 A = \dfrac{3\cos A + \cos 3A}{4}$
- $\sin^3 A = \dfrac{3\sin A - \sin 3A}{4}$

Sum-to-Product Identities

- $\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
- $\sin A - \sin B = 2\cos\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$
- $\cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
- $\cos A - \cos B = -2\sin\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$

Product-to-Sum Identities

- $\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$
- $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$
- $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

Sums of Sines and Cosines

- $A\cos x + B\sin x = \sqrt{A^2+B^2}\sin(x+\phi)$ where $\cos\phi = \dfrac{B}{\sqrt{A^2+B^2}}$ and $\sin\phi = \dfrac{A}{\sqrt{A^2+B^2}}$
- $A\cos x + B\sin x = \sqrt{A^2+B^2}\cos(x-\phi)$ where $\cos\phi = \dfrac{A}{\sqrt{A^2+B^2}}$ and $\sin\phi = \dfrac{B}{\sqrt{A^2+B^2}}$

Laws of Sines and Cosines

- $c^2 = a^2 + b^2 - 2ab\cos C$
- $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a Triangle

For a triangle with sides a, b, c and angles $\angle A$, $\angle B$, and $\angle C$,

- Area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$
- Area $= \dfrac{1}{2}ab\sin C$
- Area $= \dfrac{c^2 \sin A \sin B}{2\sin C}$

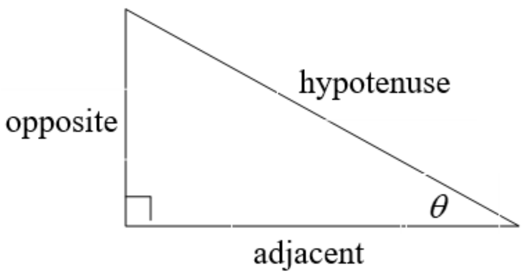
Circular Section

- Arc length: $s = r\theta$
- Area: $A = \dfrac{1}{2}r^2\theta$

Definition of the Trig Functions

Right triangle definition

For this definition we assume that $0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.



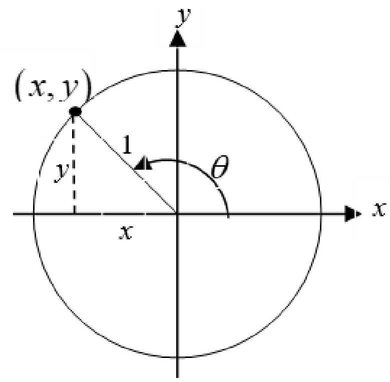
$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot(\theta) = \frac{\text{adjacent}}{\text{opposite}}$$

Unit Circle Definition

For this definition θ is any angle.



$$\sin(\theta) = \frac{y}{1} = y \qquad \csc(\theta) = \frac{1}{y}$$

$$\cos(\theta) = \frac{x}{1} = x \qquad \sec(\theta) = \frac{1}{x}$$

$$\tan(\theta) = \frac{y}{x} \qquad \cot(\theta) = \frac{x}{y}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin(\theta)$, θ can be any angle

$\cos(\theta)$, θ can be any angle

$\tan(\theta)$, $\theta \neq \left(n + \frac{1}{2}\right)\pi,\ n = 0, \pm 1, \pm 2, \ldots$

$\csc(\theta)$, $\theta \neq n\pi,\ n = 0,\ \pm 1,\ \pm 2, \ldots$

$\sec(\theta)$, $\theta \neq \left(n + \frac{1}{2}\right)\pi,\ n = 0, \pm 1, \pm 2, \ldots$

$\cot(\theta)$, $\theta \neq n\pi,\ n = 0, \pm 1, \pm 2, \ldots$

Period

The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\,\theta) \quad \rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\cos(\omega\,\theta) \quad \rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\tan(\omega\,\theta) \quad \rightarrow \quad T = \frac{\pi}{\omega}$$

$$\csc(\omega\,\theta) \quad \rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\sec(\omega\,\theta) \quad \rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\cot(\omega\,\theta) \quad \rightarrow \quad T = \frac{\pi}{\omega}$$

Range

The range is all possible values to get out of the function.

$$-1 \leq \sin(\theta) \leq 1 \qquad -1 \leq \cos(\theta) \leq 1$$

$$-\infty < \tan(\theta) < \infty \qquad -\infty < \cot(\theta) < \infty$$

$$\sec(\theta) \geq 1 \text{ and } \sec(\theta) \leq -1 \qquad \csc(\theta) \geq 1 \text{ and } \csc(\theta) \leq -1$$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Reciprocal Identities

$$\csc(\theta) = \frac{1}{\sin(\theta)} \qquad \sin(\theta) = \frac{1}{\csc(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cos(\theta) = \frac{1}{\sec(\theta)}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)} \qquad \tan(\theta) = \frac{1}{\cot(\theta)}$$

Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin(\theta) \qquad \csc(-\theta) = -\csc(\theta)$$

$$\cos(-\theta) = \cos(\theta) \qquad \sec(-\theta) = \sec(\theta)$$

$$\tan(-\theta) = -\tan(\theta) \qquad \cot(-\theta) = -\cot(\theta)$$

Periodic Formulas

If n is an integer then,

$$\sin(\theta + 2\pi n) = \sin(\theta) \qquad \csc(\theta + 2\pi n) = \csc(\theta)$$

$$\cos(\theta + 2\pi n) = \cos(\theta) \qquad \sec(\theta + 2\pi n) = \sec(\theta)$$

$$\tan(\theta + \pi n) = \tan(\theta) \qquad \cot(\theta + \pi n) = \cot(\theta)$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\begin{aligned} \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ &= 2\cos^2(\theta) - 1 \\ &= 1 - 2\sin^2(\theta) \end{aligned}$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

Half Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}}$$

Half Angle Formulas (alternate form)

$$\sin^2(\theta) = \tfrac{1}{2}\left(1 - \cos(2\theta)\right) \qquad \tan^2(\theta) = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

$$\cos^2(\theta) = \tfrac{1}{2}\left(1 + \cos(2\theta)\right)$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$$

Product to Sum Formulas

$$\sin(\alpha)\sin(\beta) = \tfrac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$

$$\cos(\alpha)\cos(\beta) = \tfrac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

$$\sin(\alpha)\cos(\beta) = \tfrac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]$$

$$\cos(\alpha)\sin(\beta) = \tfrac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right]$$

Sum to Product Formulas

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

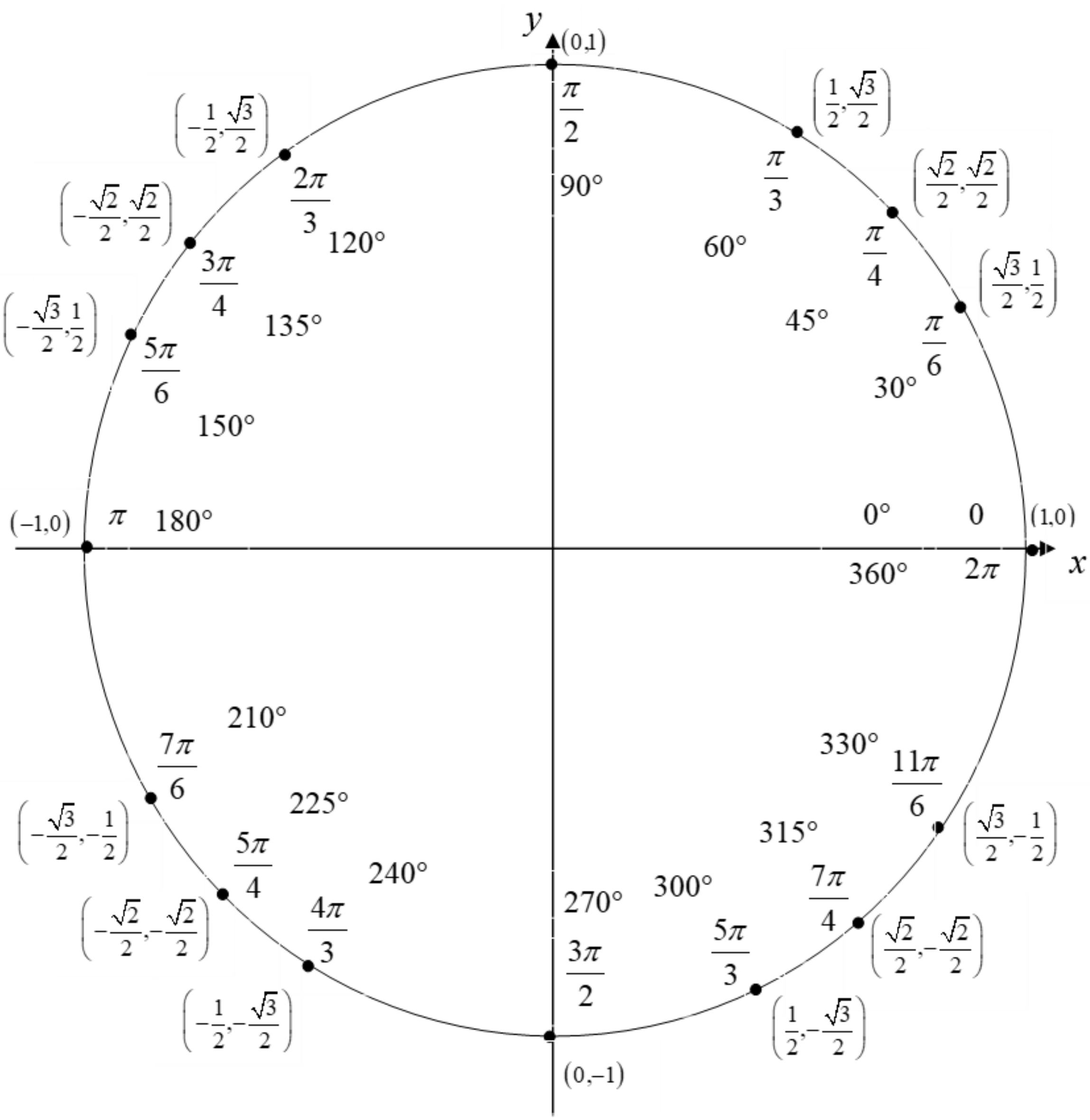
$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta) \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec(\theta) \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc(\theta)$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta) \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan(\theta)$$



For any ordered pair on the unit circle (x, y) : $\cos(\theta) = x$ and $\sin(\theta) = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \qquad\qquad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$y = \sin^{-1}(x)$ is equivalent to $x = \sin(y)$

$y = \cos^{-1}(x)$ is equivalent to $x = \cos(y)$

$y = \tan^{-1}(x)$ is equivalent to $x = \tan(y)$

Domain and Range

Function	Domain	Range
$y = \sin^{-1}(x)$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos^{-1}(x)$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan^{-1}(x)$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right) = x$ $\quad \cos^{-1}\left(\cos(\theta)\right) = \theta$

$\sin\left(\sin^{-1}(x)\right) = x$ $\quad \sin^{-1}\left(\sin(\theta)\right) = \theta$

$\tan\left(\tan^{-1}(x)\right) = x$ $\quad \tan^{-1}\left(\tan(\theta)\right) = \theta$

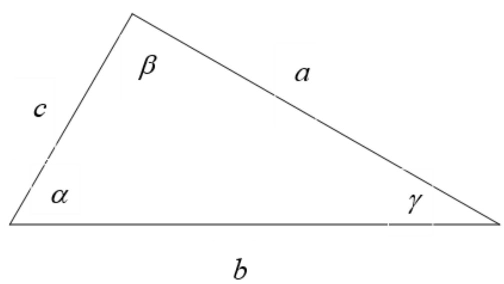
Alternate Notation

$\sin^{-1}(x) = \arcsin(x)$

$\cos^{-1}(x) = \arccos(x)$

$\tan^{-1}(x) = \arctan(x)$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$

$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos\left(\frac{1}{2}(\alpha-\beta)\right)}{\sin\left(\frac{1}{2}\gamma\right)}$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{1}{2}(\alpha-\beta)\right)}{\tan\left(\frac{1}{2}(\alpha+\beta)\right)}$$

$$\frac{b-c}{b+c} = \frac{\tan\left(\frac{1}{2}(\beta-\gamma)\right)}{\tan\left(\frac{1}{2}(\beta+\gamma)\right)}$$

$$\frac{a-c}{a+c} = \frac{\tan\left(\frac{1}{2}(\alpha-\gamma)\right)}{\tan\left(\frac{1}{2}(\alpha+\gamma)\right)}$$